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Optimal External Configuration Design of Unguided Missiles

Ömer Tanrıkulu* and Veysi Ercan†
Defense Industries Research and Development Institute, Ankara 06, Turkey

A simple optimal external configuration design method is proposed that can be used in conceptual and preliminary design stages of an unguided missile development project. Cost and constraint functions are derived from the results of linear time-invariant aeroballistic theory (constant roll rate and forward speed, decoupled axial and transverse dynamics); therefore different phases of flight are examined separately. Curvefitting is used to reduce the number of trial cases and hence the work required to obtain aerodynamic and inertial data. Optimal configurations are determined by using a modified steepest-descent algorithm. A case study is presented in which the external configuration of an unguided light assault missile is optimized for free flight at low subsonic speed. Three cost functions related to stability, range, and warhead performance, respectively, and two inequality constraint functions related directly to stability and indirectly to dispersion are considered in the case study.

Nomenclature

C_D = drag force coefficient
C_{l_p} = roll damping moment stability derivative
C_{l_δ} = roll moment due to fin cant stability derivative
C_{m_α} = static moment stability derivative
$C_{m_{\dot\alpha}}, C_{m_q}$ = transverse damping moment stability derivatives
$C_{m_{\beta p}}$ = Magnus moment stability derivative
C_{Z_α} = static force stability derivative
$\bar{d}$ = gradient vector of $F(\bar{x})$
$F(\bar{x})$ = modified cost function
$f(\bar{x})$ = cost function
$g(\bar{x})$ = inequality constraint function
$\hat{H}$ = Hessian matrix of $F(\bar{x})$
$h(\bar{x})$ = equality constraint function
I_a = axial moment of inertia, kg m^2
I_t = transverse moment of inertia, kg m^2
k_a = nondimensional axial radius of gyration
k_t = nondimensional transverse radius of gyration
m_T = total mass, kg
m_w = warhead mass, kg
p_{dyn} = Magnus instability roll rate limit, rad/s
p_{res} = yaw-pitch-roll resonance roll rate, rad/s
S = reference area, $\pi\lambda^2/4$, m^2
s_d = dynamic stability factor
s_s = static stability factor
V = speed of missile, m/s
$\bar{x}$ = vector of scaled variable geometrical parameters
$\bar{x}_l$ = lower limit of $\bar{x}$
$\bar{x}_u$ = upper limit of $\bar{x}$
α = adaptive step size of iterations
ζ = transverse damping factor
λ = reference length, rocket motor diameter, m
μ, ν = penalty constants
ρ_∞ = freestream density, kg/m^3

Introduction

THE main objectives in external configuration design of unguided missiles are to obtain adequate stability in all phases of flight, short minimum range, long maximum range, low dispersion, and large payload mass. In practice it is difficult to achieve these objectives due to the complicated nature of unguided missiles as nonlinear, time-varying, and random systems. Significant advances have been made in analysis and system identification aspects of flight dynamics of unguided missiles since World War II. A number of range, dispersion, and stability criteria have been determined by using analytical, numerical, and experimental techniques. On the other hand, very few studies have been published on external configuration design of unguided missiles.[1] The almost untouched synthesis problem is challenging for three reasons. First, design criteria are functions of aerodynamic and inertial parameters, which are in turn complicated functions of freestream flow conditions, missile geometry, and mass distribution. Second, design criteria are often contradictory. Third, design criteria are different for different flight phases of a given type of unguided missile. (Naturally, design criteria are different for different types of unguided missiles. Artillery shells, artillery missiles, high kinetic energy projectiles, light assault missiles, antitank missiles, sounding rockets, and re-entry vehicles all have to be optimized with different cost and constraint functions.)

The nature of the problem will be discussed with a simple example. Consider the unguided artillery missile configuration with cruciform tail fins presented in Fig. 1. The terms c and s denote the chord and span of tail fins, respectively, while l denotes the length of the midsection case. These are the geometrical parameters that are easiest to modify once rocket motor and warhead properties are fixed. If N different values of each of these parameters are considered, then the total number of candidate configurations to be examined becomes N^3. Changing a single geometric parameter changes all aerodynamic and inertial data. The speed of an unguided artillery missile varies significantly during flight (subsonic, transonic, and supersonic). If M different flight speeds are considered, then aerodynamic data have to be determined $M \times N^3$ times. Separate flight dynamics analysis of all of these configurations is necessary, which is difficult and costly. Moreover, such an analysis gives information about configurations with the selected discrete values of (l, c, s) only.

Consider an external configuration design analysis where only s is changed. As s is increased, m, I_a, I_t, C_{m_α}, C_{m_q}, $C_{m_{\beta p}}$, C_{l_p}, C_{l_δ}, and C_D all increase. An increase in s has both advantageous and detrimental results in terms of range, dispersion, and stability performance. Advantageous results are an increase in static and damping dynamic stability and a decrease in dispersion due to thrust misalignment. Detrimental results are a decrease in range and an increase in dispersion due to wind. An increase in s can have advantageous or detrimental results in terms of Magnus dynamic stability because I_a, I_t, C_{m_q}, $C_{m_{\beta p}}$, C_{l_p}, and C_{l_δ} all increase. An increase in s can have advantageous or detrimental results in terms of the likelihood of yaw-pitch-roll resonance because I_a, I_t, C_{m_α}, C_{l_p}, and C_{l_δ} all increase.

Also note that cost and constraint functions are different for boost and coast phases of flight of an unguided artillery missile. (As an example, most of the dispersion takes place during the boost phase.

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*Coordinator, Mechanics and Systems Engineering Research Group, PK 16, Mamak 06261. E-mail: b092136@orca.cc.metu.edu.tr. Member AIAA.
†Research Engineer, Flight Mechanics Section, Mechanics and Systems Engineering Research Group, PK 16, Mamak 06261.

Hence, dispersion criteria may not be considered among cost and constraint functions in a first-order analysis of the coast phase.)

In the second section, details of a method that can be used in the optimal external configuration design of unguided missiles will be presented. In the third section, the proposed method will be demonstrated by a case study about a light assault missile.

Optimal External Configuration Design

The method to be proposed for the optimal external configuration design has five basic steps. The first step is the selection of cost and constraint functions for different phases of flight by examining the nature of the unguided missile. The second step is the determination of aerodynamic data (at several Mach numbers depending on the flight speed range of the unguided missile) and inertial data for a sparse set of variable geometrical data by using experimental, numerical, and theoretical methods. The third step is the determination of approximate functional relationships between aerodynamic/inertial data and geometrical data by using curvefitting. This approach reduces the number of trial cases significantly, and it also creates the possibility of analyzing configurations with $\bar{x}$ data that are not included in the original sparse set. Aerodynamic data at a given Mach number and inertial data are usually smooth functions of geometrical parameters; hence, polynomial functions give satisfactory results in terms of curvefitting. The fourth step is the determination of closed-form expressions for cost, inequality constraint, and equality constraint functions by using the aerodynamic and inertial functions that were obtained by curvefitting in the third step. The fifth step is the formulation and the solution of optimization problems. One may have to consider several combinations of cost, equality constraint, and inequality constraint functions for a specified phase of flight. Hence, the number of optimization problems to be solved can be much larger than the number of flight phases under consideration. Each of these optimization problems can be stated as follows in general:

Minimize

$$f(\bar{x})$$

subject to

$$g_l(\bar{x}) \le 0 \qquad (l = 1, \ldots, N_g)$$

$$h_m(\bar{x}) = 0 \qquad (m = 1, \ldots, N_h)$$

$$\bar{x}_l \le \bar{x} \le \bar{x}_u$$

Optimal solutions are determined by using the following iterative modified steepest-descent algorithm[2]:

$$F(\bar{x}_k) = f(\bar{x}_k) + \mu_k \sum_l \{\max[g_l(\bar{x}_k), 0]\}^2 + \nu_k \sum_m \{h_m(\bar{x}_k)\}^2 \tag{1}$$

$$\bar{x}_{k+1} = \bar{x}_k - \alpha_k \bar{d}_k \tag{2}$$

$$(\bar{x}_{k+1})_j = \begin{cases} (\bar{x}_{k+1})_j, & \text{if} \quad (\bar{x}_l)_j \le (\bar{x}_{k+1})_j \le (\bar{x}_u)_j \\ (\bar{x}_l)_j, & \text{if} \quad (\bar{x}_{k+1})_j < (\bar{x}_l)_j \\ (\bar{x}_u)_j, & \text{if} \quad (\bar{x}_{k+1})_j > (\bar{x}_u)_j \end{cases}$$

$$(j = 1, 2, \ldots, n) \tag{3}$$

where

$$\alpha_k = \frac{\bar{d}_k^T \bar{d}_k}{\bar{d}_k^T \hat{H}_k \bar{d}_k} \tag{4}$$

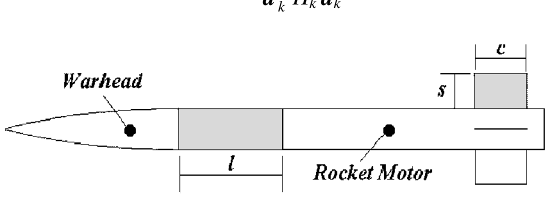

Fig. 1 Unguided artillery missile configuration.

In the preceding expressions ($k = 1, 2, 3, \ldots$) is the iteration counter; F is obtained by combining cost function and constraint functions to turn the constrained optimization problem into an unconstrained one; $\bar{x}$ has a dimension of n; and $\mu > 0$ and $\nu > 0$ are penalty constants of very large magnitude for inequality and equality constraints, respectively. Magnitudes of μ and ν are increased in every couple of steps of iteration. The term α is the adaptive step size of iterations, and for $\bar{d}$ and $\hat{H}$

$$\bar{d} = \bar{\nabla} F, \qquad d_i = \frac{\partial F}{\partial x_i} \tag{5}$$

$$\hat{H}_{ij} = \frac{\partial^2 F}{\partial x_i \partial x_j} \tag{6}$$

Case Study

The subject matter of this representative case study is the optimal external configuration design of an unguided short-range light assault missile, a schematic drawing of which is shown in Fig. 2. The one-man portable missile is shoulder launched, and there is no rocket motor propulsion once the missile leaves its tube launcher. The original warhead of the missile is to be replaced by an antipersonnel fragmentation warhead. No modifications will be performed in terms of solid propellant rocket motor because the mass of the new warhead is constrained to be less than or equal to the mass of the old one. The free flight Mach number of the missile is 0.32 with the old warhead. The missile will have a low subsonic flight speed with the new warhead as well. The missile has six planar tail fins with no dihedral, which means that it has hexagonal rotational symmetry and six planes of mirror symmetry.[3]

There are three reasons for selecting such a missile as the subject of this case study, which aims to demonstrate the proposed optimal external configuration design method as simply as possible. First, there is only one flight phase to be considered (short-duration free flight during which the missile has a relatively straight mean trajectory). Second, the flight takes place at low subsonic Mach numbers where aerodynamic characteristics do not vary much with Mach number, and hence only one set of aerodynamic data has to be determined. Third, it is possible to derive approximate closed-form expressions for a large number of aerodynamic stability derivatives (including $C_{m_{\beta p}}$) by using the method of Bryson[3] for flight at low subsonic Mach numbers.

Three geometrical parameters of the light assault missile were selected to be variable parameters of the optimization problem: span s and chord c of tail fins and length l of warhead midsection. The values that are assigned to lower and upper limits of these parameters in this case study are shown in Table 1. Five equally spaced values of each parameter were used to obtain a sparse set of 125 configurations.

Closed-form expressions that relate aerodynamic stability derivatives to geometry were obtained by using the method of Bryson,

Table 1 Upper and lower limits of variable geometrical data

Parameter	Lower limit	Upper limit
l, mm	30	60
c, mm	10	15
s, mm	50	100

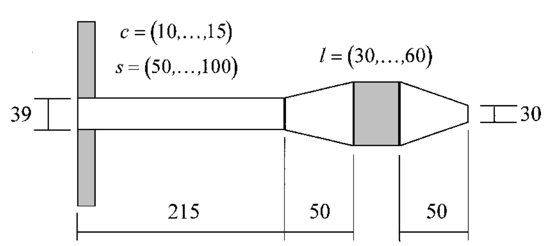

Fig. 2 Unguided light assault missile. All dimensions are in millimeters.

which is restricted to incompressible potential flow. Drag coefficient data of the 125 configurations were determined by using the Missile DATCOM database for a Mach number of 0.32. The term C_D was assumed to be related to $\bar{x}$ by the following functional relationship:

$$C_D(\bar{x}) = c_0 + c_1x_1 + c_2x_2 + c_3x_3 + c_4x_1^2 + c_5x_2^2 + c_6x_3^2 + c_7x_1x_2 + c_8x_1x_3 + c_9x_2x_3 \tag{7}$$

The $\bar{\boldsymbol{x}}$ data were scaled by using $x_1 = l/l_{\max}$, $x_2 = c/c_{\max}$, and $x_3 = s/s_{\max}$ to improve the quality of curvefitting. The c_i $(i = 0, 1, \ldots, 9)$ coefficients for C_D were determined by using the curvefitting utility of Sigma Plot software.

A Turbo Pascal program was prepared to determine inertial and aerodynamic data of the light assault missile based on a simplified solid model, the results of the method of Bryson for aerodynamic stability derivatives, and the curvefitted Missile DATCOM C_D data. Values of the following cost and inequality constraint functions were calculated for the 125 configurations by using the program.

The first cost function denoted by f_1 is related to Magnus instability and yaw-pitch-roll resonance. Linear time-invariant aeroballistic theory has two important results related to the roll rate p of a statically stable, slightly asymmetric unguided missile.[4] First, if the magnitude of p exceeds a certain limit denoted by p_{dyn} during flight, then a Magnus instability takes place. Second, if the magnitude of p coincides with a certain value denoted by p_{res}, then a yaw-pitch-roll resonance takes place. (Nonlinear aeroballistic theory predicts that a linear resonance can be followed by the lock in of p to p_{res}, which in turn can be followed by a severe instability known as catastrophic yaw due to nonlinear induced aerodynamic moments and configurational asymmetries.) Closed-form expressions for p_{dyn} and p_{res} are given next:

$$p_{\mathrm{dyn}} = \frac{2V}{\lambda}\frac{I_t}{I_a}\sqrt{\frac{M}{s_d(2 - s_d)}} \tag{8}$$

$$p_{\mathrm{res}} = (V/\lambda)\sqrt{-M} \tag{9}$$

where

$$M = \left(1/k_t^2\right)C^*_{m\alpha} \tag{10}$$

$$s_d = 2T/H \tag{11}$$

$$H = -\left[C^*_{z\alpha} + 2C^*_D + \left(1/2k_t^2\right)\left(C^*_{m_q} + C^*_{m_{\dot{\alpha}}}\right)\right] \tag{12}$$

$$T = \left(1/2k_a^2\right)C^*_{m\beta p} - C^*_{z\alpha} - C^*_D \tag{13}$$

$$k_{a,t} = \sqrt{I_{a,t}/m_T\lambda^2} \tag{14}$$

$$C^* = (\rho_\infty S\lambda/2m_T)C \tag{15}$$

The magnitude of p_{dyn} is usually three to five times larger than the magnitude of p_{res}. In external configuration design, it is desirable to have a large difference between p_{dyn} and p_{res} because one usually tries to meet the $p_{\mathrm{res}} < p < p_{\mathrm{dyn}}$ condition by using tail fin cant. Hence, the first cost function is selected as

$$f_1 = -\frac{|p_{\mathrm{dyn}} - p_{\mathrm{res}}|\lambda}{2V} \tag{16}$$

The second cost function denoted by f_2 is related to range performance. Linear time-invariant aeroballistic theory predicts the variation of speed of an unguided missile in free straight flight to be

$$V = V_0 e^{-C^*_D s} \tag{17}$$

where s is the nondimensional arc length:

$$s = \frac{1}{\lambda}\int_{t_0}^{t} V\,\mathrm{d}t \tag{18}$$

Hence, the second cost function is selected as

$$f_2 = C_D/m_T \tag{19}$$

The third cost function, denoted by f_3, is related to warhead performance. A very simple criterion in terms of weapon system effectiveness is the ratio of warhead mass m_w to total mass. Hence, the third cost function is selected as

$$f_3 = -m_w/m_T \tag{20}$$

The first inequality constraint function, denoted by g_1, is related to the static stability factor s_s, which is defined as

$$s_s = C_{m\alpha}/C_{Z\alpha} \tag{21}$$

The magnitude of s_s should be larger than a certain limiting value (which is usually taken as 1) for adequate static stability. In the case of the light assault missile, a survey of s_s data of 125 configurations showed that there were no problems in terms of static stability with $\sim 4.5 \le s_s \le \sim 5.5$. Nevertheless, the first inequality constraint function was selected as

$$g_1 = 5 - s_s \tag{22}$$

The reason for this selection is related to the dispersion of the light assault missile, which is due to aiming errors and configurational asymmetries. Nothing can be done to reduce dispersion due to aiming errors in terms of external configuration design. On the other hand, dispersion due to configurational asymmetries can be reduced by increasing s_s.

The second inequality constraint function, denoted by g_2, is related to the transverse damping factor, which is defined as

$$\zeta = H/2\sqrt{-M} \tag{23}$$

The magnitude of ζ should be larger than a certain limiting value for adequate dynamic stability. In the case of the light assault missile, a survey of ζ data of 125 configurations showed that there were no problems in terms of dynamic stability with $\sim 0.12 \le \zeta \le \sim 0.19$. Nevertheless, the second inequality constraint function was selected as

$$g_2 = 0.15 - \zeta \tag{24}$$

The reason for this selection is also related to the desire to keep dispersion due to configurational asymmetries below a certain level.

Polynomial functional relationships similar to Eq. (7) were assumed to exist between each f, g, and $\bar{\boldsymbol{x}}$, coefficients of which were determined by using the curvefitting utility of Sigma Plot software. Each cost function was scaled in such a way that the difference between its maximum and minimum values and its mean value were both equal to 1. The optimal external configuration for each cost function was determined separately by using a Turbo Pascal program implementing the modified steepest-descent algorithm that was discussed in the preceding section. [Four different optimization problems were solved with f_i $(i = 1, 2, 3)$, $f_4 = f_1 + f_2 + f_3$, $g_1 \le 0$, $g_2 \le 0$, and $\bar{\boldsymbol{x}}_l \le \bar{\boldsymbol{x}} \le \bar{\boldsymbol{x}}_u$.] Optimal $\bar{\boldsymbol{x}}$ values for roll dynamic stability performance (f_1), range performance (f_2), warhead performance (f_3), and composite cost function (f_4) are presented in Table 2 with the corresponding values of the cost and inequality constraint functions.

The variations of f_1, g_1, and g_2 with x_2 and x_3 at $x_1 = 0.5$ are shown in Figs. 3–5. The variations of f_2, f_3, f_4, g_1, and g_2 with x_2 and x_3 at $x_1 = 1$ are shown in Figs. 6–10. Figure 3 shows that the lowest f_1 value is located at the upper-left-hand corner of the f_1 vs x_2 and x_3 graph at $x_1 = 0.5$. Figures 4 and 5 show that inequality constraints $g_1 \le 0$ and $g_2 \le 0$ are both satisfied at that particular point. Similar visual inspection of Figs. 6–10 and an examination of the cost and constraint function data of the 125 configurations have shown the authors that correct optimal solutions were found for all four of the optimization problems. No convergence problems were observed in iterations with different initial conditions, which resulted in the same optimal solutions.

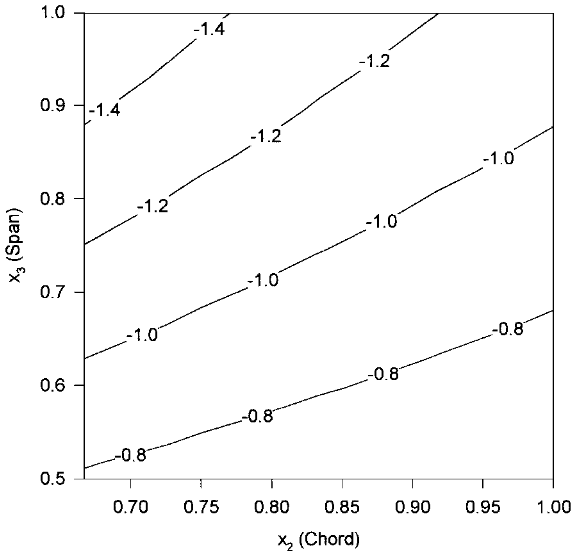

Fig. 3 Variation of f_1 cost function with x_2 and x_3 for x_1 = 0.5; f_1 (Magnus instability resonance).

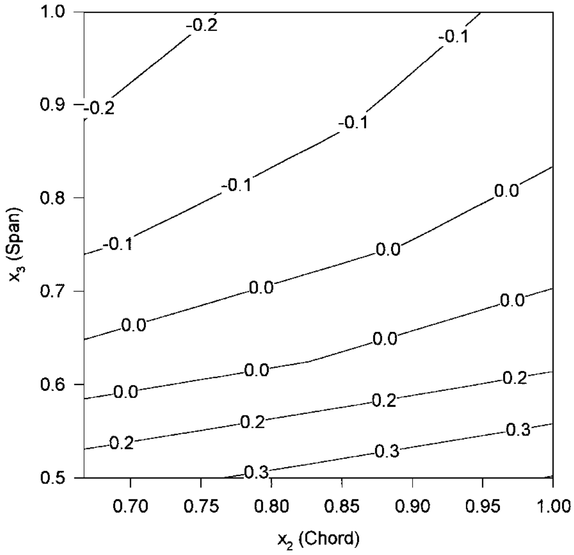

Fig. 4 Variation of g_1 inequality constraint function with x_2 and x_3 for x_1 = 0.5; g_1 (static stability).

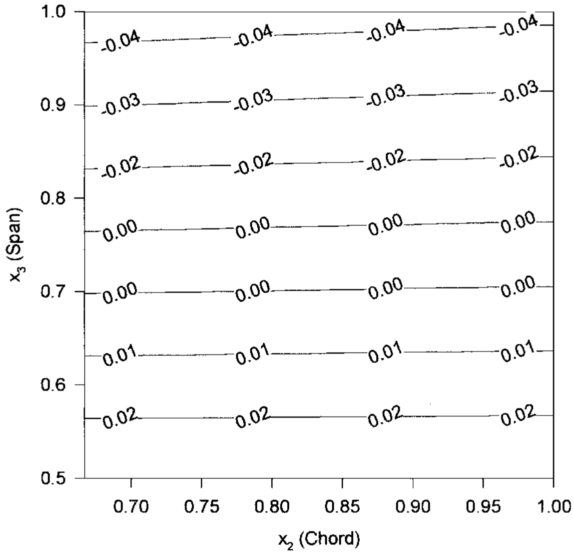

Fig. 5 Variation of g_2 inequality constraint function with x_2 and x_3 for x_1 = 0.5; g_2 (damping dynamic stability).

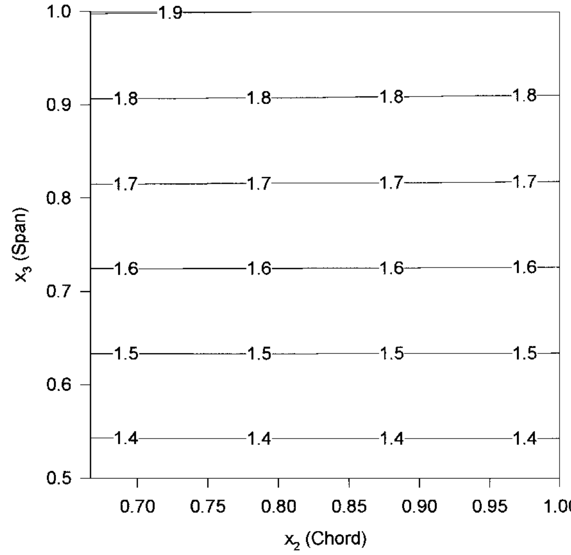

Fig. 6 Variation of f_2 cost function with x_2 and x_3 for x_1 = 1.0; f_2 (range).

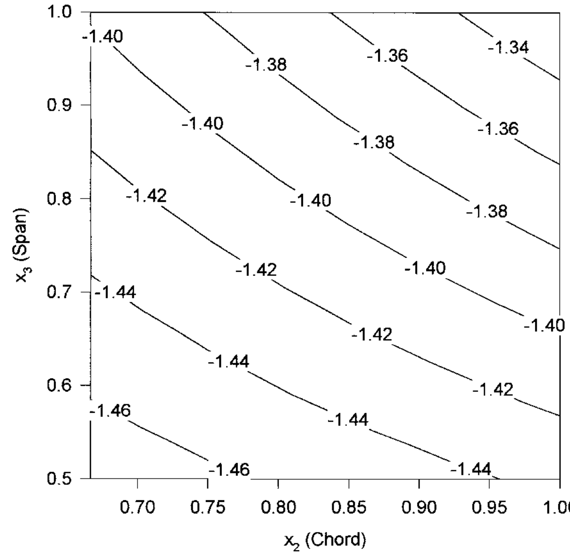

Fig. 7 Variation of f_3 cost function with x_2 and x_3 for x_1 = 1.0; f_3 (warhead).

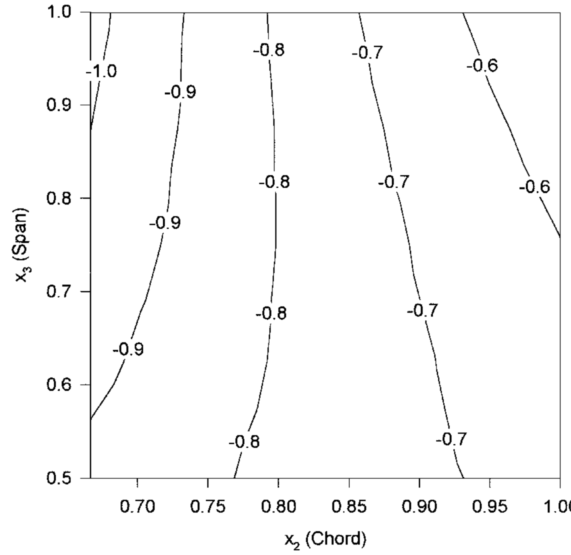

Fig. 8 Variation of f_4 cost function with x_2 and x_3 for x_1 = 1.0; f_4 (composite).

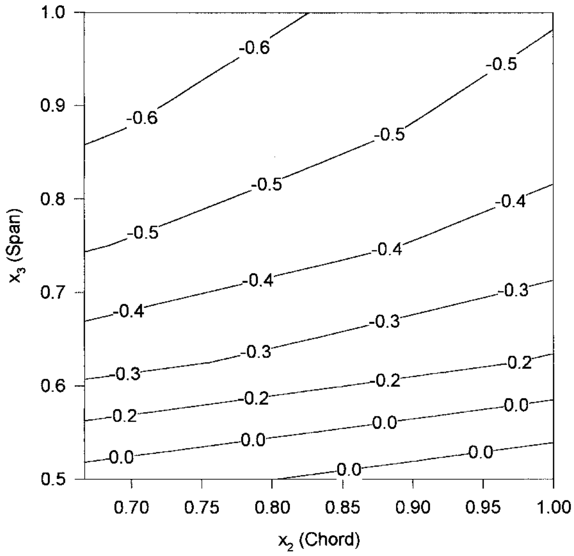

Fig. 9 Variation of g_1 inequality constraint function with x_2 and x_3 for x_1 = 1.0; g_1 (static stability).

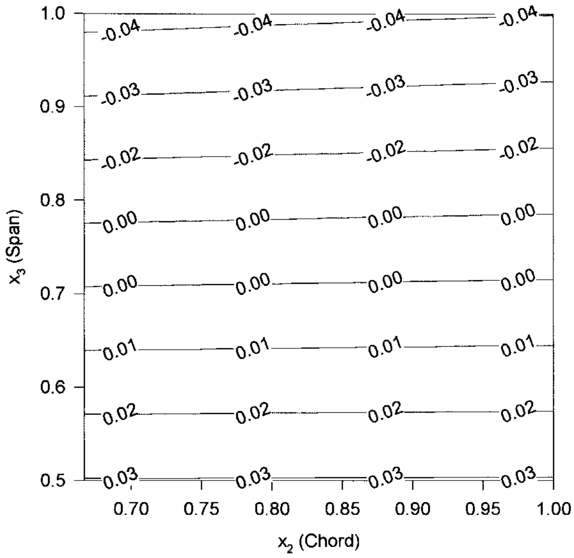

Fig. 10 Variation of g_2 inequality constraint function with x_2 and x_3 for x_1 = 1.0; g_2 (damping dynamic stability).

Table 2 Optimal $\bar{x}$ values for f_i (i = 1, 2, 3, 4)

$f_1 = -1.570$	$f_2 = -1.592$	$f_3 = -1.437$	$f_4 = -1.021$
$g_1 = -0.239$	$g_1 = -0.363$	$g_1 = -0.446$	$g_1 = -0.682$
$g_2 = -0.045$	$g_2 = 0.000$	$g_2 = 0.000$	$g_2 = -0.043$
$\bar{x}_{\text{opt}} = \begin{Bmatrix} 0.500 \\ 0.667 \\ 1.000 \end{Bmatrix}$	$\bar{x}_{\text{opt}} = \begin{Bmatrix} 0.996 \\ 0.856 \\ 0.712 \end{Bmatrix}$	$\bar{x}_{\text{opt}} = \begin{Bmatrix} 0.997 \\ 0.667 \\ 0.707 \end{Bmatrix}$	$\bar{x}_{\text{opt}} = \begin{Bmatrix} 1.000 \\ 0.667 \\ 1.000 \end{Bmatrix}$

Conclusion

In this paper an optimal external configuration design method for unguided missiles was presented. The method is based on linear time-invariant aeroballistic theory, and hence different phases of flight are analyzed separately. Cost and constraint functions are determined by using curvefitted aerodynamic and inertial functions based on a sparse set of geometrical data. Optimal external configurations are obtained by using a modified steepest-descentalgorithm. The method was demonstrated through a representative case study based on a light assault missile with subsonic free flight. Different cost functions that were subject to inequality constraints were minimized successfully in a three-dimensional geometrical parameter space. The method can be used as a simple and reliable tool in the preliminary design stage of an unguided missile development project.

There have been significant advances in the analysis and system identification of unguided missiles since World War II. On the other hand, the external configuration design of unguided missiles has been neglected despite its obvious importance. Hence, a number of problems that require further investigation remain: optimal external configuration design based on nonlinear time-invariant aeroballistic theory and the development of a method that can determine the best configuration for the whole of a specified mission rather than the best configurations for different phases of the mission.

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J. R. Maus
Associate Editor